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Résumé :

Une méthode acoustique est proposée pour mesurer la résistivité au passage de l'air et l'épaisseur d'un échantillon poreux à structure rigide. Les méthodes classiques [14,16,17] permettant la mesure de la résistivité (où la perméabilité visqueuse) nécessitent la connaissance préalable de la porosité. La méthode présentée dans ce travail est basée sur un modèle temporel du problème direct dans lequel une expression simplifiée (indépendante de la fréquence et de la porosité) du coefficient de transmission dans le régime de Darcy (très basses fréquences) est établie. Cette expression ne dépend que de la perméabilité visqueuse (où la résistivité au passage de l'air) et de l'épaisseur d'un échantillon poreux. Le problème inverse est résolu en minimisant, au sens des moindres carrés entre le signal transmis théorique et expérimentale, permettant ainsi la détermination de l'épaisseur et de la perméabilité visqueuse (où la résistivité) de deux échantillons de même mousse en plastique avec des épaisseurs différentes. Cette méthode présente l'avantage d'être simple, rapide et efficace.

Abstract:

An acoustic method is proposed for measuring the flow resistivity and the thickness of air-saturated porous materials. The conventional methods [14, 16, 17] for the measurement of the flow resistivity (or the viscous permeability) require the prior knowledge of the porosity. The method presented in this work is based on a temporal model of the direct problem in which a simplified expression (independent of frequency and porosity) of the transmission coefficient at the Darcy's regime (low frequency range) is established, this expression depends only on the viscous permeability (or the flow resistivity) and the thickness of a porous sample. The inverse problem is solved based on the least-square numerical method using experimental transmitted wave in time domain. Tests are performed using two samples of different thicknesses to same industrial plastic foam, thereby enabling the determination the thickness and flow resistivity of foam plastic. This method has the advantage of being simple, fast and efficient.

Keywords: Characterization, porous medium, rigid frame, Darcy's regime.

1 Introduction

The acoustic characterization of porous materials saturated by air [4,5] such as plastic foams, fibrous, or granular materials is of great interest for a wide range of industrial applications. These materials are frequently used in the automotive and aeronautics industries and in the building trade. One important parameter that appears in theories of sound propagation in porous materials at a low-frequency range [6,7] is the specific flow resistivity σ[8-9]. This parameter is defined as the ratio between the pressure difference across a sample and the velocity of flow of air through that sample per unit cube; the flows being considered are steady and non pulsating. The permeability $k0$ is related to the specific flow resistivity σ by the relation $k0 = \eta/\sigma$, where η is the fluid viscosity. Several methods [10-11] have been developed in the past to measure the flow resistivity. Among these methods, we distinguish between the so-called direct methods [10- 12] which do not use sound waves, and indirect methods [13 -11] that use sound waves transmitted or reflected by the porous material. The practical implementation of the direct methods could be both complex and expensive. Most of the acoustic (indirect) methods [10-11] require a priori estimation of the porosity, or other non-acoustic parameters [14-15] (tortuosity, viscous and thermal characteristic lengths, thermal permeability). The proposed procedure is an indirect acoustical method for measuring the flow resistivity and the thickness of porous materials saturated by air (and therefore flow resistivity), without knowing in advance the porosity or other non-acoustic setting but just using experimental transmitted waves at low frequency.

2 Model

In the acoustics of porous materials, one distinguishes two situations according to whether the frame is moving or not. In the first case, the dynamics of the waves due to the coupling between the solid skeleton and the fluid is well described by the Biot theory [3]. In air-saturated porous media the structure is generally motionless and the waves propagate only in the fluid. This case is described by the model of equivalent fluid [7], which is a particular case of the Biot model, in which the interactions between the fluid and the structure are taken into account in two frequency response factors: the dynamic tortuosity of the medium α(ω) given by Johnson *et al.*[2] and the dynamic compressibility of the fluid included in the porous material β(ω) given by Allard [1], (ω is the pulsation frequency). In the frequency domain, these factors multiply the density of the fluid and its compressibility, respectively, and represent the deviation from the behavior of the fluid in free space as the frequency changes.

Consider a homogeneous porous material that occupies the region $0 \le x \le L$. A sound pulse impinges normally on the medium. It generates an acoustic pressure field $p(x, t)$ and an acoustic velocity field $v(x, t)$ within the material. The acoustic fields satisfy the Euler equation and the constitutive equation (along the x axis):

$$\rho\alpha(\omega)j\omega v = \frac{\partial p}{\partial x}, \qquad \frac{\beta(\omega)}{K_a}j\omega p = \frac{\partial v}{\partial x} \tag{1}$$

Where $j^2 = -1$, ρ is the suturing fluid density, and K_a is the compressibility modulus of the fluid.

The expression of a pressure wave incident plane, unit amplitude, arriving at normal incidence to the porous material is given by

$$p^i(x,t) = e^{-j(kx-\omega t)}, \tag{2}$$

where $k = \frac{\omega}{c_0} = \omega\sqrt{\frac{\rho}{K_a}}$, k is the wave number of the free fluid. In the medium (I) (x < 0), the movements results from the superposition of incident and reflect waves:

$$p_1(x,t) = e^{-j(kx-\omega t)} + R(\omega)e^{-j(-kx-\omega t)} \quad (3)$$

where $R(\omega)$ is the reflection coefficient.
According to Eq. (1), the expression of the velocity field in the medium (I) wrote:

$$v_1(x,t) = \frac{1}{Z_f}\left(e^{-j(kx-\omega t)} - R(\omega)e^{-j(-kx-\omega t)}\right) \quad (4)$$

Where $Z_f = \sqrt{\rho K_a}$
In the medium (II) corresponding to the porous material, the expression of the pressure and velocity field are:

$$p_2(x,t) = A(\omega)e^{-j(kx-\omega t)} + B(\omega)e^{-j(-kx-\omega t)} \quad (5)$$

$$v_2(x,t) = \frac{1}{Z(\omega)}\left(A(\omega)e^{-j(kx-\omega t)} - B(\omega)e^{-j(-kx-\omega t)}\right) \quad (6)$$

In these expression $A(\omega)$ and $B(\omega)$ are function of pulsation for determining, $Z(\omega)$ and $k(\omega)$ are the characteristic impedance and the wave number, respectively, of the acoustic wave in the porous medium. These are two complex quantities:

$$k(\omega) = \omega\sqrt{\frac{\rho\alpha(\omega)\beta(\omega)}{K_a}}, \qquad Z(\omega) = \sqrt{\frac{\rho K_a \alpha(\omega)}{\beta(\omega)}} \quad (7)$$

Finally, in the medium (III), the expression of the pressure and velocity fields of the wave transmitted through the porous material are

$$p_3(x,t) = T(\omega)e^{-j(k(x-L)-\omega t)}, \quad (8)$$

$$v_3(x,t) = \frac{1}{Z_f}T(\omega)e^{-j(k(x-L)-\omega t)} \quad (9)$$

where $T(\omega)$ is the transmission coefficient.
To derive the transmission scattering operator, it is assumed that the pressure field and flow velocity are continuous at the material boundary:

$$p_1(0^+,\omega) = p_2(0^-,\omega), \quad (10)$$
$$p_2(L^-,\omega) = p_3(L^+,\omega), \quad (11)$$
$$v_1(0^-,\omega) = \phi v_2(0^+,\omega), \quad (12)$$
$$\phi v_2(L^-,\omega) = v_3(L^+,\omega), \quad (13)$$

where ϕ is the porosity of the medium and the ± superscript denotes the limit from right and left, respectively.
Using the relation (10)-(13), we obtain the transmission coefficient of a slab of porous material given by :

$$T(\omega) = \frac{2Y(\omega)}{2Y(\omega)\cosh(jk(\omega)L) + \left(1+Y^2(\omega)\right)\sinh(jk(\omega)L)} \quad (14)$$

where

$$Y(\omega) = \phi\sqrt{\frac{\beta(\omega)}{\alpha(\omega)}} \quad \text{and} \quad k(\omega) = \omega\sqrt{\frac{\rho\alpha(\omega)\beta(\omega)}{K_a}} \tag{15}$$

In the Darcy's regime [7, 18] (low-frequency approximation), the expressions of the responses factors α(ω) and β(ω) when ω⟶0 are given by the relations [7]

$$\alpha(\omega) = \frac{\eta\phi}{\rho k_0 j\omega}, \qquad \beta(\omega) = \gamma \tag{16}$$

where k0 is the static permeability, ϕ is the porosity, and γ is the adiabatic constant.
Using expressions (16) of the dynamic tortuosity and compressibility, we obtain the following expression for the transmission coefficient:

$$T(\omega) = \frac{2C_1\sqrt{j\omega}}{2C_1\sqrt{j\omega}cosh\left(LC_2\sqrt{j\omega}\right)+\left(1+C_1^2 j\omega\right)sinh\left(LC_2\sqrt{j\omega}\right)} \tag{17}$$

where

$$C_1 = \sqrt{\frac{\gamma\rho k_0\phi}{\eta}}, \qquad C_2 = \sqrt{\frac{\gamma\eta\phi}{K_a k_0}} \tag{18}$$

By doing the Taylor series expansion of the transmission coefficient, when the frequency tends to zero (ω→0), we obtain:

$$T(\omega) = \left(\frac{1}{1+\frac{LC_2}{2C_1}}\right)\left[1 - j\frac{\omega}{\omega_c} + \cdots\right] \tag{19}$$

where,

$$\omega_c = \frac{2\left(1+\frac{LC_2}{2C_1}\right)}{LC_1C_2\left(1+\frac{LC_2}{C_1}+\frac{1}{6}\left(\frac{LC_2}{C_1}\right)^2\right)} \tag{20}$$

As a first approximation, in the very low frequencies, the transmission coefficient is given by the first term

$$T(\omega) = \frac{1}{1+\frac{LC_2}{2C_1}} = \frac{1}{1+\frac{L}{2k_0}\frac{\eta}{\sqrt{\rho K_a}}} \tag{21}$$

This simplified expression of the coefficient of transmission is independent of the frequency and the porosity of the material, and depends only on the static permeability and thickness of the material.
The incident $p^i(t)$ and transmitted $p^t(t)$ fields are related in time domain by the transmission scattering operator $T(\omega)$ [14,17],

$$\begin{aligned} p^t(x,t) &= T(t) * p^i(t) \\ p^t(x,t) &= \int_0^t T(\tau)\, p^i\left(t - \tau - \frac{(x-L)}{c_0}\right) d\tau \end{aligned} \tag{22}$$

The temporal operator kernel $T(t)$ is calculated by taking the inverse Fourier transform of the transmission coefficient of slab of porous material given by:

$$T(t) = \frac{1}{1+\frac{LC_2}{2C_1}}\delta(t) \tag{23}$$

3 Inverse of experimental data

The inverse problem is to fond values for parameters, flow resistivity σ and thickness L, that minimizes the functions:

$$U(\sigma) = \int_0^t \left[p_{exp}^t(x,t) - p^t(x,t)\right]^2 dt, \tag{24}$$

and

$$U(L) = \int_0^t \left[p_{exp}^t(x,t) - p^t(x,t)\right]^2 dt, \tag{25}$$

where $p_{exp}^t(x,t)$ is the determined transmitted signal and $p^t(x,t)$ is the transmitted wave predict from Eq. (22). The inverse problem is solved numerically by the least-square method.

Experiments are performed in a guide (pipe), having a diameter of 5 cm and of length 50m. This length has been chosen for the propagation of transient signals at low frequency. It is not important to keep the pipe straight; it can be rolled in order to save space without perturbations on experimental signals (the cutoff frequency of the tube *fc* ~ 4 kHz). A sound source Driver unit “Brand” constituted by loudspeaker Realistic 40-9000 is used. Bursts are provided by synthesized function generator Stanford Research Systems model DS345-30 MHz. The signals are amplified and filtered using model SR 650-Dual channel filter, Standford Research Systems. The signals (incident and transmitted) are measured using the same microphone (Bruel&Kjaer, 4190) in the same position in the tube. The incident signal is measured without a porous sample; however, the transmitted signal is measured with the porous sample. The experimental setup is shown in Fig. 1.
Consider two cylindrical sample M1 and M2 of the same plastic foam M of diameter 5cm, and thickness L1=10.1cm and L2=20.2cm respectively, sample M was characterized using classical methods [10] given σ = 6500 ±500 $Nm^{-4}s$. figures 2(a) and 2(b) show the experimental incident signal (solid line) generated by the loudspeaker in the frequency bandwidth (40-100)Hz, and the experimental transmitted signal (dashed line) of the two samples M1 and M2.

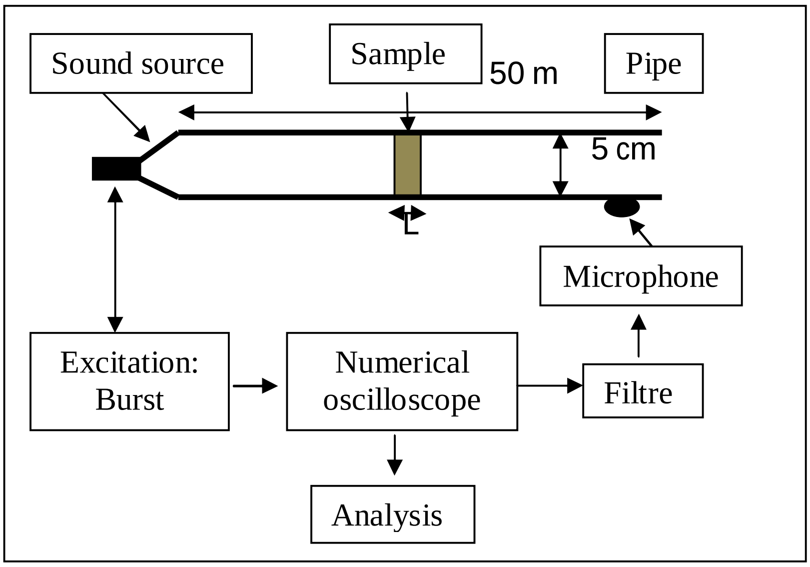

Fig. 1. Experimental setup of acoustic measurements.

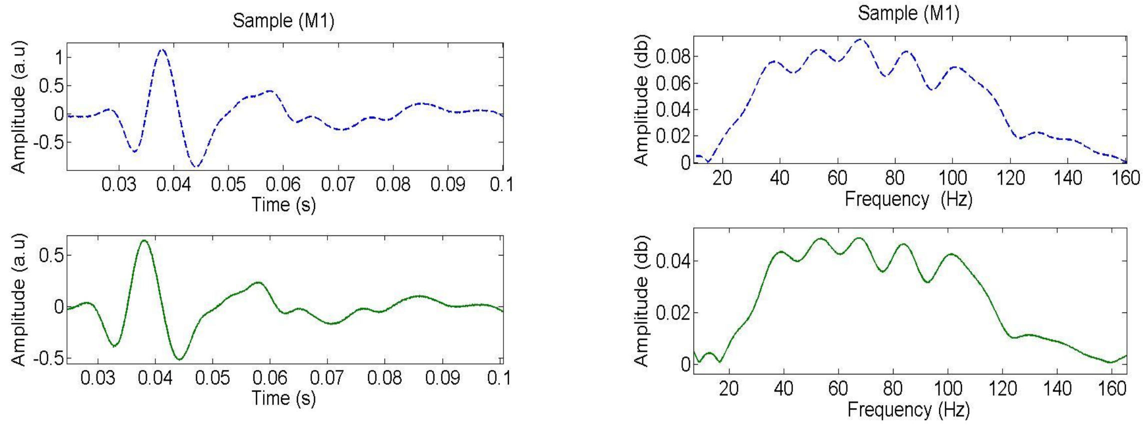

Fig. 2(a) – Experimental incident signal (dashed line) and experimental transmitted signal (solid line) at left and their spectra at right of the sample M1.

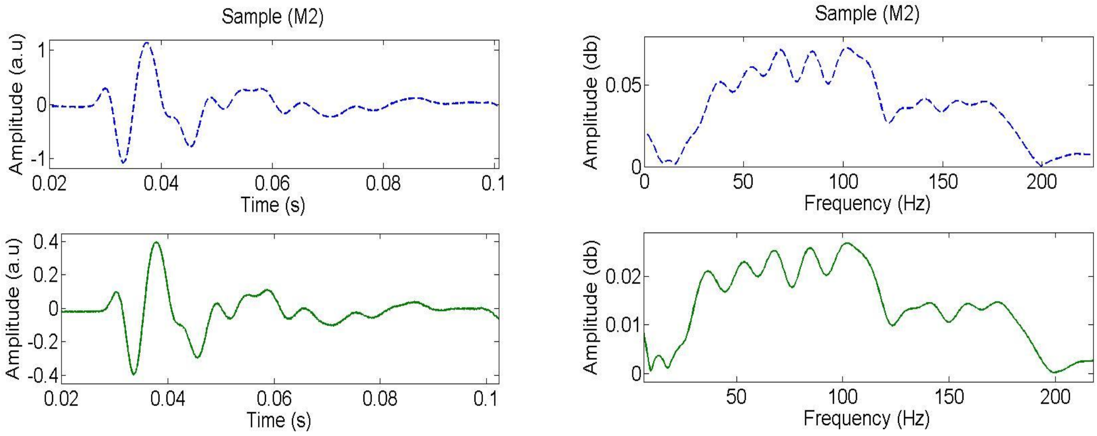

Fig. 2(b) – Experimental incident signal (dashed line) and experimental transmitted signal (solid line) at left and their spectra at right of the sample M2.

In the first case we propose to determine the flow resistivity σ1 and σ2 of both sample M1 and M2 of thickness L1 = 10.1cm and L2 = 20.2cm respectively assumed unknown. Different frequency bandwidth have been investigated between (50 – 100)Hz. The experimental incident signals generated by the loudspeaker (solid line) and the transmitted one (dashed line) and their spectra are given in Fig.2; we can see in this case that the center frequency of the signal is between 60 and 80Hz. By solving the inverse problem for the flow resistivity and minimizing the cost function U given by Eq.(23), the obtained optimized values of the flow resistivity are given by the table 1. The reader can see the slight difference between the optimized values of the flow resistivity obtained with this method and the other classical method (Bies and Hansen [10]).

We show the result of the inverse problem in Figs.4.Using these optimized values; we compare the simulated transmitted signals and experimental signals. The results of the comparison are shown in Figs.5. The correspondence between experiment and theory is good, which leads us to conclude that this method based on the solution of the inverse problem is appropriate for estimating the flow resistivity of porous materials with rigid frame.

Fréquency (Hz)	40-70	60-80	70-100	Average
σ1 (10^{+3} $Nm^{-4}s$)	6.40	6.51	7.00	6.63±0.30
σ2 (10^{+3} $Nm^{-4}s$)	6.51	7.00	6.75	6.75±0.25

Table 1 – Characteristics of samples M1 and M2 obtained by solving the inverse problem for the resistivity σ

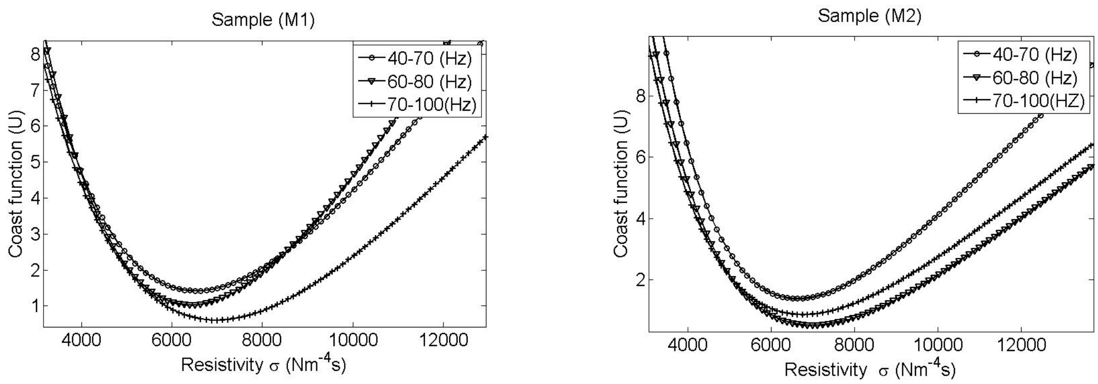

Figure 4 – Variation of the minimization function U with resistivity σ of samples M1 and M2

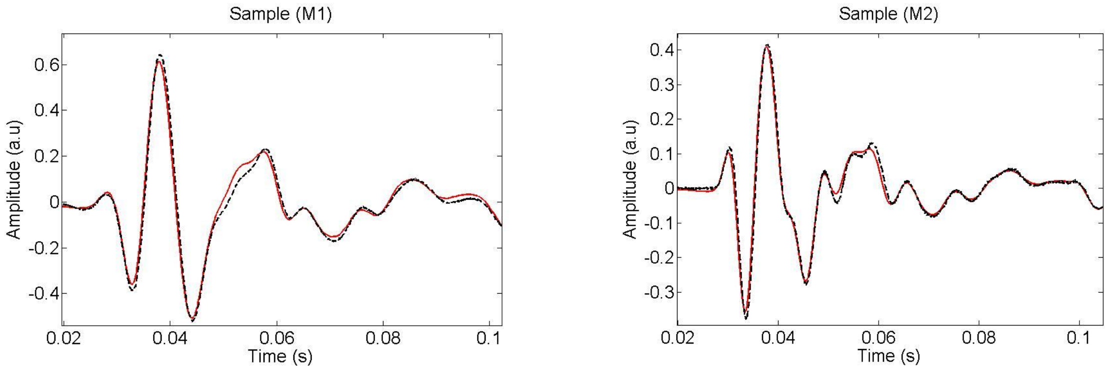

Figure 5. – (Color online) Comparison between the experimental transmitted signal (dashed line) and the simulated transmitted signals (solid line) using the reconstructed values of σ of samples M1 and M2

Let us now, in the second case, solve the inverse problem for measuring the thickness of the samples M1 and M2, assumed unknown, in the same frequency bandwidth of (40-100)Hz, the flow resistivity is fixed to σ = 6500 Nms^{-4} . By solving the inverse problem and minimizing the cost function U given by Eq.(25) we obtain the following optimized values of the thickness of both samples M1 and M2 given by the table .2.

Fréquency (Hz)	40-70	60-80	70-100	Average
L1 (cm)	9.79	9.79	10.37	09.98±0.29
L2 (cm)	20.04	20.09	20.33	20.15±0.15

Tableau 2 – Characteristics of samples M1 and M2 obtained by solving the inverse problem for the Thickness L

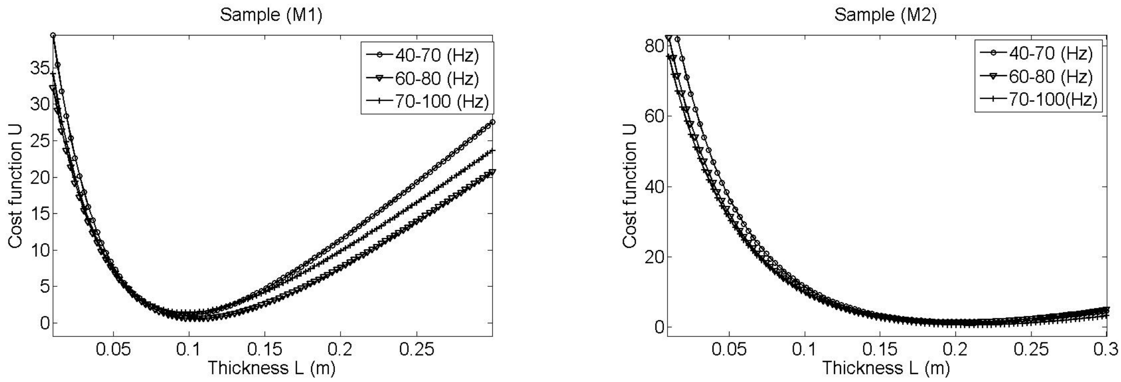

Figure 7 - Variation of the minimization function U with thickness L of samples M1 and M2

We show the result of the inverse problem in Figs. 7. It can be seen that for the different low frequency bandwidths of the experimental incident signals, the optimized values obtained using this method are close to those produced using classical methods [14,17,10,19]. The results of the inversion for the flow resistivity and the thickness are slightly different and those given by other methods [14,17,10,19] . The difference between the optimized values and experimental values does not exceed 4%.This study has been carried on, in the frequency bandwidth of 100-200 Hz, and has also given good results. This simple method seems to be effective for measuring the flow resistivity or the thickness of the porous material saturated with air and offers another faster and simpler alternative to conventional methods

4 Conclusion

In this article, an inverse scattering estimate of flow resistivity and thickness was given by solving the inverse problem for waves transmitted by a slab of air-saturated porous material. The inverse problem is solved numerically by the least-square method. The reconstructed values of flow resistivity and thickness are close to those using classical method. The important result in this study is that is now possible, with the simplified expression of the transmitted coefficient, to measure the flow resistivity and thickness, without knowing the porosity or any other parameters of the materials, and just by using the experimental transmitted wave at low frequencies.

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